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Double SQUID tunable flux qubit manipulated by fast pulses: operation requirements, dissipation and decoherence

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Abstract. A double SQUID manipulated by fast magnetic flux pulses can be used as a tunable flux qubit. In this paper we study the requirements for the qubit operation and evaluate the dissipation and decoherence due to the manipulation, with particular attention to the contribution related to the applied tuning control, not present in simpler flux qubits. Furthermore, we shortly discuss the possibility to use an integrated Rapid Single Flux Quantum logic for the qubit control.

PACS. 03.67.Lx Quantum computation – 85.25.Dq Superconducting quantum interference devices (SQUIDs)

1 Introduction

Quantum computing can overcome the limitations that are intrinsic and unavoidable in classical instruments; moreover it is a formidable framework for the study and understanding of quantum mechanics [1]. Different qubits (the basic elements of a quantum computer) based on solid state superconducting devices have been realized and tested, individually and in simple coupled configurations [2–15]. Their coherent manipulation, generally performed by NMR-like microwave excitations or by fast pulses, is a critical question: it must be fast, reliable, simple and easily integrable. In this context one of the most interesting possible strategies consists in the use of an integrated Rapid Single Flux Quantum (RSFQ) logic for qubit manipulation [16–18].

In this paper we consider a particular qubit based on a Superconducting Quantum Interference Device (SQUID) with a high degree of tunability, the so called double SQUID [19–23], and its manipulation performed by fast variation of the magnetic flux controls (instead of the usually considered microwave excitation). The behaviour of a typical device is studied in order to define its operating parameters, to fix the requirement for the manipulating pulses, and to evaluate dissipation and decoherence due to the manipulation; in particular we consider the contribution due to the extra control necessary for the qubit tuning, not present in other flux qubits, such as the persistent current qubit [4] or the simple rf SQUID qubit. Finally we shortly consider the possibility to use the RSFQ logic with appropriate modifications in order to perform qubit manipulation.

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2 The tunable flux qubit

A rf SQUID consists of a superconducting loop of inductance L, interrupted by a Josephson junction of critical current I_0 and capacitance C, and biased by an applied magnetic flux Φ_x. For appropriate conditions it can be effectively used as a flux qubit. A more tunable device, the double SQUID [19,20], is obtained by replacing the single junction with a dc SQUID, a smaller superconducting loop of inductance ℓ interrupted by two identical junctions with critical current J and capacitance C_0, biased by an applied magnetic flux Φ_c (Fig. 1a). For $\ell \ll \varphi_0/J$ (where $\Phi_0 \cong 2.07 \times 10^{-15}$ Wb is the flux quantum and $\varphi_0 = \Phi_0/2\pi$ is the reduced flux quantum) the dc SQUID behaves approximately as a single junction with total capacitance $C = 2C_0$ and tunable critical current $I_0 = 2\,J\cos(\pi\Phi_c/\Phi_0)$, and the double SQUID can be used as a tunable rf SQUID. The system dynamics is described by the canonical variable φ (the phase difference across the dc SQUID, related to the flux Φ in the large loop by $\varphi = \Phi/\varphi_0$), and by the relative conjugate variable $p = -i\hbar\partial/\partial\varphi$, with Hamiltonian:

$$H = \frac{p^2}{2M} + E_L\left[\frac{1}{2}(\varphi-\varphi_x)^2 - \beta\cos(\varphi)\right] \qquad (1)$$

where $E_L = \varphi_0^2/L$ is the energy scale, $\varphi_x = \Phi_x/\varphi_0$ and $\varphi_c = \Phi_c/\varphi_0$ are the reduced control fluxes, $M = C\varphi_0^2$ is the effective mass, and $\beta = 2JL/\varphi_0\cos(\varphi_c/2)$.

For $\varphi_x = \pi$ (corresponding to $\Phi_x = \Phi_0/2$) the potential is symmetric, with two identical minima separated by a barrier if it $1 < \beta < 4.60$ (Fig. 1b). In this case the energy spectrum is characterized by a degenerate situation, with the first two levels separated by an energy gap

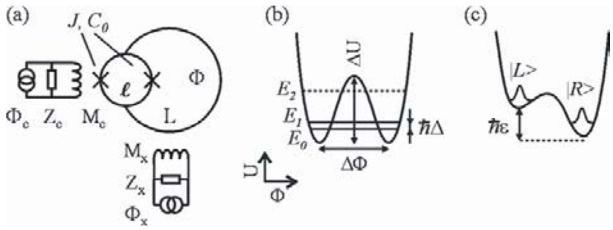

Fig. 1. (a) Scheme of the double SQUID with the two independent control coils. (b) Potential of the double SQUID in the symmetric case, and relative energy levels. (c) Potential in the asymmetric case.

$\hbar\Delta = E_1 - E_0$ which it is smaller than the separation from upper levels ($E_2 - E_1 \gg \hbar\Delta$). In the absence of possible excitations to these upper levels (due for example to temperature or to nonadiabatic variations) a two state approximation can be used by considering the reduced energy basis with just the first two energy eigenstates $|0\rangle$ and $|1\rangle$. A second basis can be introduced, consisting of the two flux states centred in the left and right minima respectively, with approximately $|L\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|R\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. Also in the asymmetric case, for φ_x different but close to π, one can again use the two state approximation; now the Hamiltonian (1) in the flux basis can be rewritten as follows:

$$H_{flux} = -\frac{\hbar\Delta}{2}\sigma_x - \frac{\hbar\varepsilon}{2}\sigma_z \tag{2}$$

where $\sigma_x, \sigma_y, \sigma_z$ are the standard Pauli matrices, and $\hbar\varepsilon$ is the energy separation between the two minima (potential asymmetry) (Fig. 1c). The eigenstates of Hamiltonian (2) can be written as $|\tilde{0}\rangle = c|L\rangle + s|R\rangle$ and $|\tilde{1}\rangle = s|L\rangle - c|R\rangle$ (we use here tilded kets to avoid confusion with the energy eigenstates in the symmetric case $|0\rangle$ and $|1\rangle$), with $c = \cos(\theta/2)$, $s = \sin(\theta/2)$, and $\theta = \arctan(\Delta/\varepsilon)$, while the energy gap between these states is $\hbar\Omega = \hbar\sqrt{\Delta^2 + \varepsilon^2}$. The energy eigenstates $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ are equivalent to $|0\rangle$ and $|1\rangle$ only in the symmetric case, when $\varepsilon = 0$ and therefore $\theta = \pi/2$.

The considered system can be used as a qubit by mapping the computational qubit states "0" and "1" in, for example, the two distinct flux states $|L\rangle$ and $|R\rangle$. The possibility of tuning the parameterΔ, generally fixed in other types of superconducting qubits, allows a complete control of the qubit and justifies the name "tunable qubit" used for this system. NMR-like manipulation with microwave pulses can be performed, but now a complete manipulation is also possible, just with fast flux pulses. For example, the state preparation can be done by strongly unbalancing the potential with the control Φ_x in order to have just one minimum (for example the left one), then waiting for a time lapse sufficient for relaxation to this minimum, and finally returning to the symmetric situation still maintaining the barrier high. This ensures an initial preparation in the state $|L\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, that remains frozen because of the high barrier. If the barrier is lowered, a coherent evolution starts, and after a time t the state evolves in $|\psi(t)\rangle = \cos(\hbar\Delta t/2)|L\rangle - i\sin(\hbar\Delta t/2)|R\rangle$. Finally the barrier is raised again and the new state is frozen. Other kinds of manipulations can be performed with similar sequences of variations, allowing the full control of a single qubit [21,22]. More qubits can be coupled together by using superconducting tunable switching [23], allowing in this way a controlled entanglement between them.

3 Operation requirements

In this paragraph we study, both numerically and with analytical approximations, the behaviour of the tunable flux qubit in order to derive the requirements on the control pulses for both Φ_x and Φ_c. A generic scheme with typical qubit parameters is considered, so that the following results can be easily applied to different configurations.

In the symmetric case some important quantities can be defined: the distance $\Delta\varphi$ between the minima, the barrier height ΔU, and the small oscillation frequency in the minima $\omega_b = \sqrt{(dU^2/d^2\varphi)_{\min}/M}$ (Fig. 1b). The system dynamics is interesting just for β greater and close to 1, so that these parameters can be expanded in series for $0 < \beta - 1 \ll 1$ in order to derive approximated analytical expressions. In order to extend their validity in the range $0 < \beta < 2$ we multiply these expressions by appropriate powers of β that are determined empirically by comparison with the analytical results, under the requirement of a maximum relative discrepancy below 10^{-3}, obtaining:

$$\begin{aligned} \Delta\varphi &\cong \sqrt{24\,(\beta-1)}\,\beta^{-0.36} \\ \Delta U &\cong \frac{3}{2}E_L\,(\beta-1)^2\,\beta^{-0.82} \\ \omega_b &\cong \omega_L\sqrt{2\,(\beta-1)}\,\beta^{-0.145} \end{aligned} \tag{3}$$

where $\omega_L = 1/\sqrt{LC}$. The parameter Δ can be evaluated by using an approximated expression in the limit $\Delta U \gg \hbar\omega_b$ [24]:

$$\Delta \cong A\,\omega_b\sqrt{B\frac{\Delta U}{\hbar\omega_b}}\exp\left(-B\frac{\Delta U}{2\hbar\omega_b}\right) \tag{4}$$

with $A \cong 1$ and $B \cong 10.2$. In a more general case, considering also a slightly asymmetric potential with energy unbalancing $\hbar\varepsilon = E_L(\varphi_x - \pi)\Delta\varphi$ (Fig. 1c), we have $E_1 - E_0 = \hbar\Omega = \hbar\sqrt{\Delta^2 + \varepsilon^2}$, and the spacing $E_2 - E_1$ can be roughly estimated by $\hbar(\omega_b - \Omega)$. We have compared these analytical approximated results with numerical simulations obtained by solving the time independent Schrödinger equation with Hamiltonian (1) using standard numerical techniques. Let us consider a realistic case by choosing a set of typical parameters for the double SQUID: large loop inductance $L = 85$ pH, small loop inductance $\ell < 5$ pH, single junction critical current $J = 5$ μA and capacitance $C_0 = 0.25$ pF. In Figure 2 we plot the level spacing $\Delta/2\pi = (E_1 - E_0)/h$ in the symmetric case as a function of the flux control Φ_c, obtained both analytically (lower straight line) and numerically (lower dashed line), and the spacing to upper levels $(E_2 - E_1)/h$, again

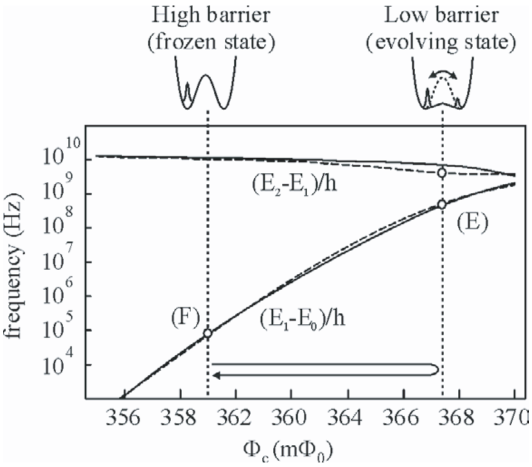

Fig. 2. Energy levels spacing $(E_1 - E_0)/h$ (lower curves) and $(E_2 - E_1)/h$ (upper curves), obtained both analytically (straight lines) and numerically (dashed lines) in the symmetric case.

obtained both analytically (upper straight line) and numerically (upper dashed line). These curves can be used to fix the fundamental requirements on the control pulse Φ_c. Manipulations are performed by switching the system between two distinct working points: point (F) where the barrier is very high and the state is frozen, and point (E) where the barrier is low and the free evolution of the state occurs. We choose (F) and (E) in order to have $\Delta_F/2\pi \approx 100$ kHz and $\Delta_E/2\pi = 500$ MHz respectively, obtaining $\Phi_{cF} = 359.3\,\mathrm{m}\Phi_0$ and $\Phi_{cE} = 367.6\,\mathrm{m}\Phi_0$, corresponding to a pulse amplitude $\Delta\Phi_c = 8.3\,\mathrm{m}\Phi_0$. The rise/fall time τ of this pulse must be chosen with some attention. In fact the variation rate must be fast with respect to the free evolution frequency $\Delta/2\pi = 500$ MHz, but at the same time it must not excite upper non computational states, and so it must also be smaller than $(E_2 - E_1)/h \approx 4$ GHz (in the point E, the worst case), so that it must be 90 ps $< \tau <$ 700 ps.

In order to have a complete preparation in the left (or right) state it is necessary to reduce to zero the coefficient $c^2 = \left|\langle L|\,\tilde{0}\rangle\right|^2 = (1+\varepsilon/\Omega)/2$ (or $s^2 = \left|\langle R|\,\tilde{0}\rangle\right|^2 = (1-\varepsilon/\Omega)/2$). This expression can be calculated: we obtain that when $\Phi_c = \Phi_{cE}$, it is necessary to have a variation of Φ_x from the symmetric point $\Phi_0/2$ larger than $\Delta\Phi_x \approx 0.2\,\mathrm{m}\Phi_0$ in order to reduce the coefficient below 1%. The rise/fall times of the Φ_x pulse must undergo the same requirements as the Φ_c pulse.

4 Dissipation and decoherence

For the purposes of this work we limit our analysis to the study of decoherence contributions due to both the manipulating fluxes Φ_x and Φ_c, and the relative bias circuitry (for the study of other contributions such as intrinsic dissipation, flux fluctuations and readout effects see, for example, Refs. [25–28]). The flux controls are applied by using superconducting coils of inductance L_k (the index $k = x, c$ identifies which of the two controls are being considered) coupled with mutual inductance M_k to the considered qubit loop (Fig. 1a). Each coil is biased by a circuit that we model with an ideal current source I_k in parallel to a frequency dependent complex impedance $Z_k(\omega)$, and to the related generator of current noise δI_k with spectral density $S_{\delta I_k} = 4k_b\tilde{T}_k/Re(Z_k)$, where $\tilde{T}_k(\omega) = \hbar\omega/2k_b \coth(\hbar\omega/2k_bT)$ is the effective temperature. The current noise δI_k causes a corresponding flux noise $\delta\Phi_k = M_k\delta I_k$, and a consequent fluctuation of the parameters Δ and ε. For small noise contributions, in linear approximation, we can assume $\delta\,\Delta = (d\Delta/d\Phi_c)\,\delta\,\Phi_c$ and $\delta\,\varepsilon = (d\varepsilon/d\Phi_x)\,\delta\,\Phi_x$ respectively, with spectral densities that can be rearranged in the following expressions:

$$S_{\delta\Delta}(\omega) = \frac{4k_b\tilde{T}_c}{\hbar}\frac{R_0}{R_c}\left(\frac{M_c}{L}\right)^2\left[\frac{\partial(\hbar\Delta/E_L)}{\partial(\Phi_c/\varphi_0)}\right]^2$$

$$S_{\delta\varepsilon}(\omega) = \frac{4k_b\tilde{T}_x}{\hbar}\frac{R_0}{R_x}\left(\frac{M_x}{L}\right)^2\left[\frac{\partial(\hbar\varepsilon/E_L)}{\partial(\Phi_x/\varphi_0)}\right]^2 \tag{5}$$

with $R_0 = \varphi_0^2/\hbar \cong 1026\Omega$ and $R_k = Re\left[Z_k(\omega)\right]$. The noise contributions can be added in Hamiltonian (2) giving:

$$H_{flux} = \left(-\frac{\hbar\Delta}{2}\sigma_x - \frac{\hbar\varepsilon}{2}\sigma_z\right) - \hbar\,\delta\Delta\,\sigma_x - \hbar\,\delta\varepsilon\,\sigma_z. \tag{6}$$

In the energy basis of the noiseless Hamiltonian these contributions can be reorganized in a longitudinal and in a transverse component ($\tilde{\sigma}_x$ and $\tilde{\sigma}_z$ are the Pauli matrices related to the energy basis):

$$H_{energy} = -\frac{\hbar\Omega}{2}\tilde{\sigma}_z - \hbar\left(\delta\varepsilon\frac{\varepsilon}{\Omega} + \delta\Delta\frac{\Delta}{\Omega}\right)\tilde{\sigma}_z - \hbar\left(\delta\varepsilon\frac{\Delta}{\Omega} - \delta\Delta\frac{\varepsilon}{\Omega}\right)\tilde{\sigma}_x. \tag{7}$$

The two state system theory for small perturbing noise can be simply extended to the case of two distinct uncorrelated noise sources with the form of equation (7), giving simple expressions for the relaxation rate Γ_1, the pure dephasing rate Γ_φ and the dephasing rate Γ_2 [29]:

$$\Gamma_1 = \left(\frac{\Delta}{\Omega}\right)^2 S_{\delta\varepsilon}(\Omega) + \left(\frac{\varepsilon}{\Omega}\right)^2 S_{\delta\Delta}(\Omega)$$

$$\Gamma_\varphi = \left(\frac{\varepsilon}{\Omega}\right)^2 S_{\delta\varepsilon}(0) + \left(\frac{\Delta}{\Omega}\right)^2 S_{\delta\Delta}(0)$$

$$\Gamma_2 = \frac{1}{2}\Gamma_1 + \Gamma_\varphi. \tag{8}$$

We notice from equations (5) and (8) that relaxation and decoherence depend quadratically on the coupling strengths (M_k/L_k) and only linearly on the effective temperatures and on the dissipating contributions R_k/R_0. This gives an important guideline for the design of the qubit control, indicating the convenience of a strong decoupling from the bias coils.

Let us consider typical control current pulses with amplitude of the order of 10 μA. In order to generate the

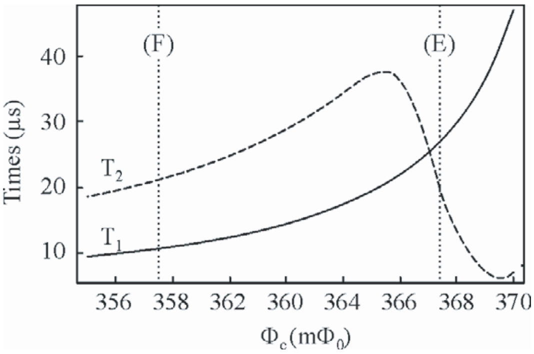

Fig. 3. Relaxation and dephasing times T_1 and T_2 as a function of the control flux Φ_c in the symmetric case.

required flux pulses $\Delta\Phi_x = 0.2$ mΦ_0 and $\Delta\Phi_c = 8.3\,$mΦ_0 we need couplings $M_x = 41$ fH and $M_c = 1.7$ pH.

For a first rough evaluation we assume $R_k/T = 5\ \Omega$/mK (for example $R_k = 100\,\Omega$ at $T = 20$ mK) at any frequency and for both bias lines. With these assumptions and by using equation (8) we can plot $T_1 = \Gamma_1^{-1}$ (straight line) and $T_2 = \Gamma_2^{-1}$ (dashed line) in the symmetric case as a function of Φ_c (Fig. 3). The obtained values T_1 and T_2, well above 10 μs during all possible manipulations, must be compared with the typical time required for one operation (in our case below 2 ns), so that it results, from this rough evaluation, that the considered system and manipulation procedure is in principle adequate for quantum computing applications. This result can be simply scaled for different assumptions by using equations (5), and it is also possible to introduce more complex, not flat noise spectral densities.

5 RSFQ manipulation

RSFQ logic is an architecture based on resistively shunted Josephson elements, allowing ultra-fast digital operations [30]. It can be simply and effectively used in order to produce the flux pulses necessary for the qubit manipulation described in this paper. For example, in a simple and effective scheme [16–18] the switching of the flux state in a RSFQ flip-flop can be used as the flux pulse for the control of a qubit inductively coupled by a superconducting transformer (Fig. 4).

All the required devices, qubits and RSFQ controls, can be realized in the same chip using the same technology, with great advantage for the simplicity and integrability of the system, but one can also use a coupled-chip design if necessary.

The integration of the flux qubit with RSFQ controls is in principle compatible with the operation requirements discussed in this paper, but the direct use of a conventional logic presents some problems. First of all, since the RSFQ logic requires resistive shunts, the typical impedance seen by the qubit would be very small, of the order of few Ohms or less, with consequent reduction of times T_1 and T_2. Second, typical RSFQ pulse rise/fall times are extremely

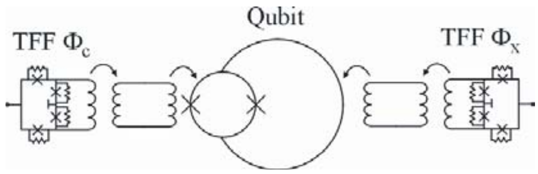

Fig. 4. Scheme of two RSFQ T-flip flop used as pulse generators for the control of the tunable flux qubit.

short, of the order of tens of picoseconds. If directly applied to the qubit, these signals would induce transitions to non computational states. Finally, a standard RSFQ circuitry is designed to work at liquid Helium temperature (4.2 K), but for quantum computing applications lower temperatures (10 mK) are needed. These problems could be solved by developing an unconventional RSFQ logic (for example based on non-linear shunts with very high impedance in the rest condition that decreases in the presence of a SFQ pulse [31]) together with an appropriate on-chip filtering of the transmitted signal and an optimization of the qubit parameters.

Different efforts are starting in this direction, in particular in the frame of the UE project "RSFQubit", and first prototypes of chips with an RSFQ flip-flop coupled to a tunable flux qubit are under fabrication. The first results will give important indications for the future developments.

6 Conclusions

A double SQUID with a flux pulse control scheme can be effectively used as a tunable flux qubit. In this paper we have studied the requirements for the control fluxes, in particular for the tuning bias control, both with approximated analytical expressions and with numerical simulations. For a system with typical parameters we have obtained required control pulses amplitudes $\Delta\Phi_x \approx 0.2$ mΦ_0 and $\Delta\Phi_c \approx 8$ mΦ_0, with rise/fall times τ that must be in the limit 90 ps $< \tau <$ 700 ps. Relaxation and decoherence times (just due to the manipulating controls) are expected to be well above 10 μs if one supposes an effective dissipation of 100 Ω at 20 mK, which is suitable for quantum computing applications.

RSFQ logic is an interesting candidate for the control of this qubit, provided the development is performed of an unconventional RSFQ design that would accomplish the qubit requirements. Possible future work concerns the optimization of the parameters for the considered system, the development and test of an RSFQ qubit control, and the realization of a final experiment for the observation of coherent manipulation with pulses in a tunable flux qubit.

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